3.2 derivatives 2014-10-08 day 31

16.) $y = \frac{1}{x+1}$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)+1} - \frac{1}{x+1}}{\Delta x} \qquad \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)+1}\left(\frac{x+1}{x+1}\right) - \frac{1}{x+1}\left(\frac{x+\Delta x+1}{x+\Delta x+1}\right)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\frac{x+1}{(x+1)(x+\Delta x+1)} - \frac{(x+\Delta x+1)}{(x+1)(x+\Delta x+1)}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\frac{-\Delta x}{(x+1)(x+\Delta x+1)}}{\Delta x} \qquad \lim_{\Delta x \to 0} \frac{-\Delta x}{(x+1)(x+\Delta x+1)}\left(\frac{1}{\Delta x}\right)$$

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x} \cdot 1}{\cancel{\Delta x}(x+1)(x+\Delta x+1)} \qquad \lim_{\Delta x \to 0} \frac{1}{(x+1)(x+\Delta x+1)}$$

$$\lim_{\Delta x \to 0} \frac{-1}{(x+1)(x+\Delta x+1)} \qquad \cancel{x^2+\Delta x x+x+x+\Delta x+1}$$

$$\lim_{\Delta x \to 0} \frac{-1}{(x+1)(x+0+1)} \qquad \lim_{\Delta x \to 0} \frac{-1}{(x+1)(x+1)}$$

$$\frac{dy}{dx} = \frac{-1}{(x+1)(x+1)}$$

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22) $$\lim_{w \to x} \frac{\frac{4}{3}\pi w^3 - \frac{4}{3}\pi x^3}{w-x} =$$

$$\lim_{w \to x} \frac{\frac{4}{3}\pi(w^3 - x^3)}{w-x} =$$

$$\lim_{w \to x} \frac{4\pi}{3} \frac{\cancel{(w-x)}(w^2 + wx + x^2)}{\cancel{(w-x)}} =$$

$$\lim_{w \to x} \frac{4\pi}{3}(w^2 + wx + x^2) =$$

$$\frac{4\pi}{3}(w^2 + wx + x^2) = \frac{4\pi}{3}(x^2 + xx + x^2) =$$

$$\frac{4\pi}{3}(3x^2) = \frac{12\pi x^2}{3} = \boxed{4\pi x^2}$$

3.2 derivatives

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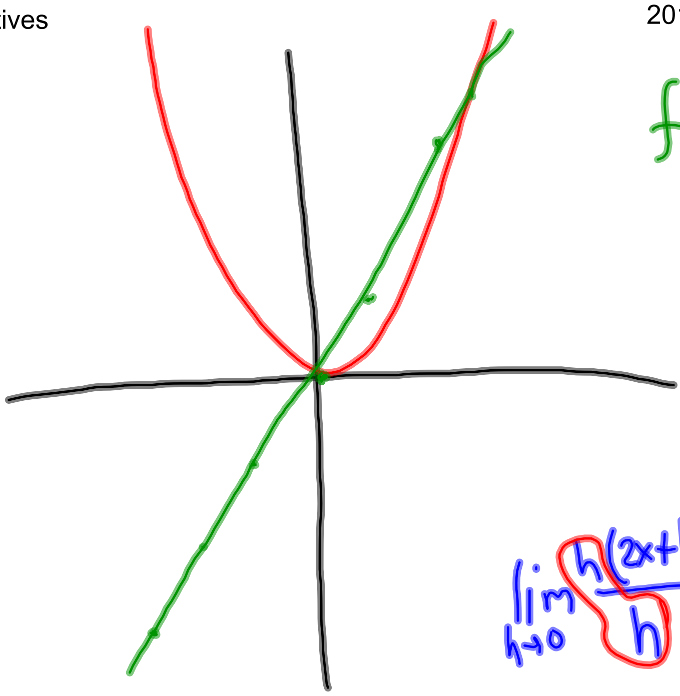

$$f(x)=x^2$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$$

$$\lim_{h\to 0}\frac{h(2x+h)}{h}=\lim_{h\to 0}\frac{2xh+h^2}{h}$$

$$=\lim_{h\to 0}2x+h=2x$$

3.2 derivatives

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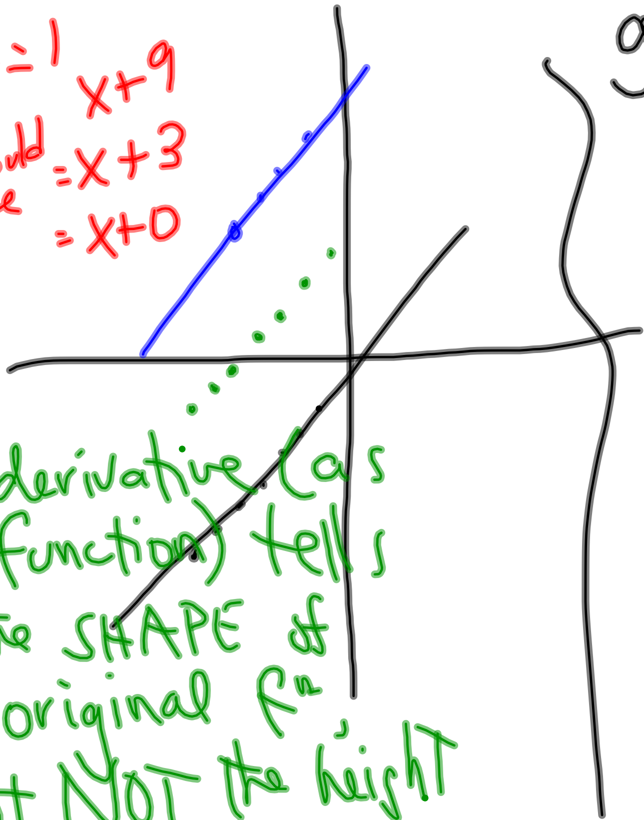
$f'(x)=1$

$f(x)$ could be $x+9$

$=x+3$

$=x+0$



The derivative (as a function) tells me the SHAPE of the original fn but NOT the height

guess: $f'(c)=0$

$f(x)=c$

$$f'(c)=\lim_{w\to x}\frac{f(w)-f(x)}{w-x}$$

$$=\lim_{w\to x}\frac{c-c}{w-x}$$

$$=\lim_{w\to x}\frac{0}{w-x}=0$$

3.2 derivatives 2014-10-08 day 31

Remember A function is differentiable on an interval (a,b)

quantifiers
$\forall$
$\exists$

$\Longrightarrow$ ["implies"] the f^n is continuous on interval (a,b).

If p then q

$p \Longrightarrow q$ "implies"

p = "my name is BOB"
q = "I am a boy"

Not necessarily true

① If $\neg p$ then $\neg q$

"not" negation "not"

② if q then p
converse

③ if $\neg q$ then $\neg p$
contra-positive
must be true

"two way facts"
an original statement is true, its converse is true.

$p \Longleftrightarrow q$
p iff q

3.2 derivatives

defined on all real #5

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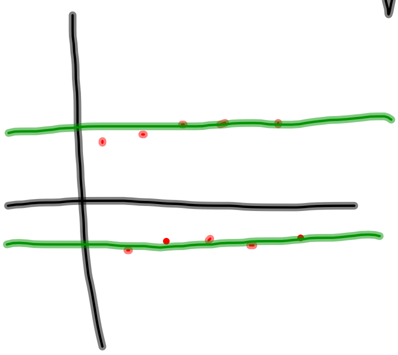
Is there a function that is
Nowhere continuous
and therefore
nowhere differentiable?



$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a rational number } \left[\frac{p}{q},\ p, q \text{ are integers},\ q \neq 0\right] \\ -1 & \text{if } x \text{ is an irrational number} \end{cases}$$